GCSE Mathematics

Problem Solving Questions

Algebra

Foundation

Problems 1 to 10

Contents

Algebra – Foundation – Problems 1 to 10

Each of these problems is presented in four different ways depending on how you want to use them in the classroom.

Section 1

Problems & Hints

This section contains the problems on a worksheet with a hints box to help guide the student through the problem.

Section 2

Problems & Mixed up Hints

The section is similar to the last except the hints are provided in a random order and the students have to sort the steps out first.

Section 3

Just the Problems

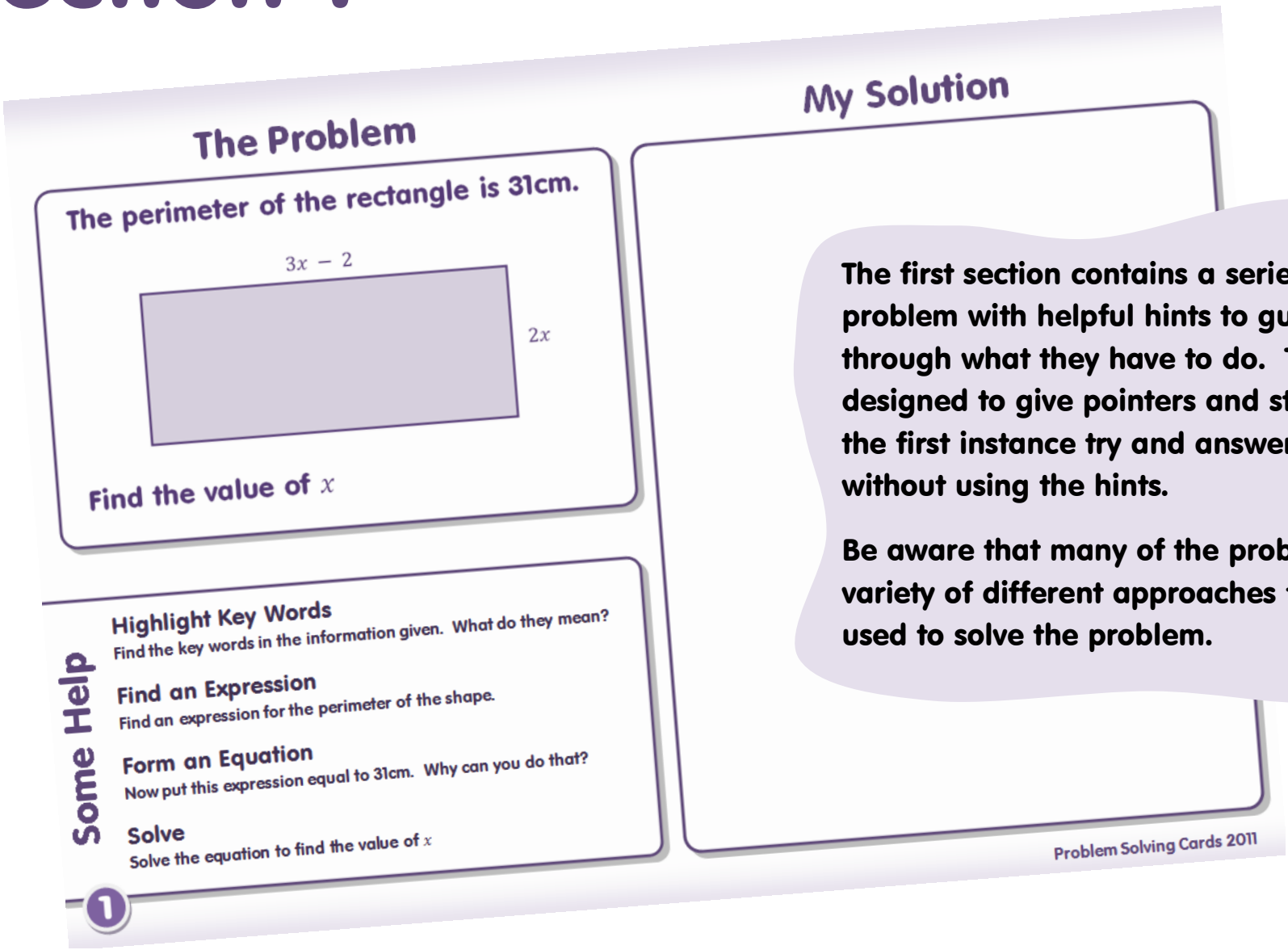
These are just the problems, with a solution box for the students working.

Section 4

Worksheet Variations

An additional 20 worksheets that are full of questions similar to the problems opposite. These can be used for additional practice.

Section 1



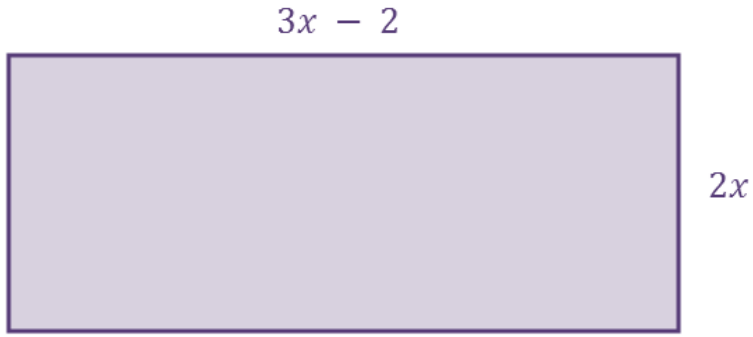
The first section contains a series of multistep problem with helpful hints to guide students through what they have to do. The hints are designed to give pointers and students should in the first instance try and answer the problem without using the hints.

Be aware that many of the problems have a variety of different approaches that could be used to solve the problem.

Problem & Hints

The Problem

The perimeter of the rectangle is 31cm.



Find the value of x

My Solution

Some Help

Highlight Key Words

Find the key words in the information given. What do they mean?

Find an Expression

Find an expression for the perimeter of the shape.

Form an Equation

Now put this expression equal to 31cm. Why can you do that?

Solve

Solve the equation to find the value of x

The Problem

Every **row** of the following square adds up to 18

a	a	a
2b	a	a
a	b	2c

Work out the value of a + b + c

My Solution

Some Help

Highlight the Key Information

What is the key piece of information you are told?

Work out a using the Top Row

What do you know the top row adds up to? What do you notice about the expressions in each of the boxes on this row? Now work out the value of a from this

Now work out b and c

Use your value of a and what you know the 2nd row adds up to to work out the value of b? Use the 3rd row to work out c.

Substitute

Substitute your values of a, b and c into the expression given

The Problem

Here are two cards with expressions on them.

ab	$2a + b - c$

$a = 5$ and $b = 3$

Find the value of c so that the two cards are equal.

Some Help

Highlight the Key Information

What are the key pieces of information that you are given?

Substitute

Substitute the values of a and b into the expression on the first card. What does this tell you that the second card must equal?

Substitute Again

Substitute the values of a and b into the second expression. Can you work out the value of c now to make the cards equal?

My Solution

The Problem

Adam, Barry and Charlie are brothers.

Here is some information about their ages

- Adam is twice as old as Barry
- Charlie is three years younger than Barry
- The sum of all their ages is 53

How old is Barry?

My Solution

Some Help

Highlight the Key Information

What are the key pieces of information that you are given?

Introduce some Algebra

Assume that Barry is b years old. Can you find expressions for the age of his two brothers Adam and Charlie in terms of b?

Find an Expression

Find an expression for the sum of the brothers' ages.

Form an Equation and Solve

Put this expression equal to 53. Why can you do this? Now solve your equation.

The Problem

The first three diagrams in a sequence are shown.

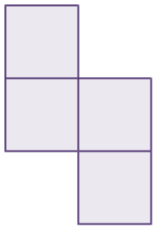


Diagram 1

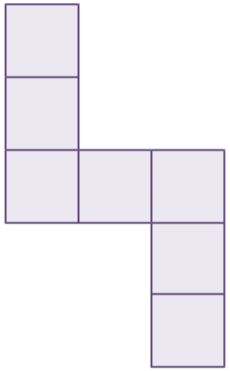


Diagram 2

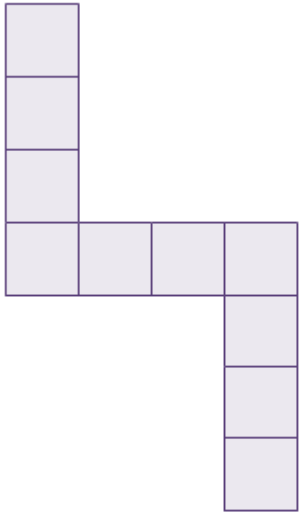


Diagram 3

How many squares will there be in Diagram 120?

My Solution

Some Help

Put your Results into a Table

Make a table for your results. For each diagram write down how many squares make up the shape.

Spot a Pattern

Use your table to spot the pattern.
Can you find an expression for the n^{th} term of your pattern?

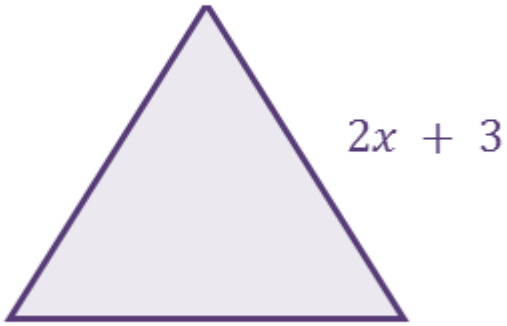
Substitute

Use your expression for the n^{th} term to find out how many squares will be needed for diagram 120. Think what you need to do with the number 120.

The Problem

The diagram shows an equilateral triangle.

The perimeter of the triangle is 42cm.



Find the value of x?

Some Help

Highlight Key Words

Find the key words in the information given. What do they mean?

Find an Expression

Find an expression for the perimeter of the shape.

Form an Equation

Now put this expression equal to 42cm. Why can you do that?

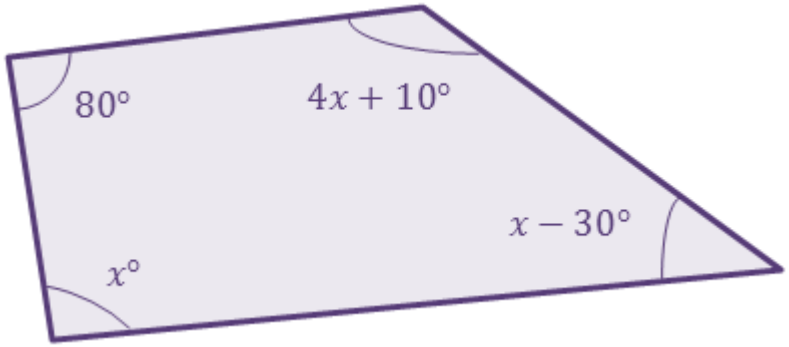
Solve

Solve the equation to find the value of x

My Solution

The Problem

The angles in a quadrilateral are shown.



Find the size of the largest angle.

My Solution

Some Help

Find an Expression

Find an expression for the sum of all the angles.

Form an Equation

What can you put this expression equal to? Think about what the angles in a quadrilateral add up to.

Solve your Equation

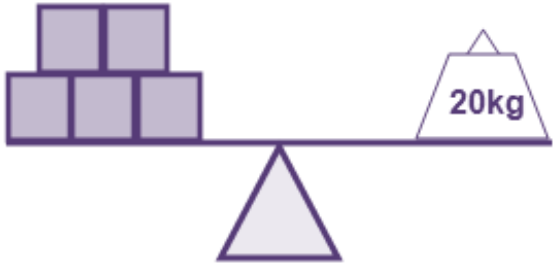
Solve the equation to find the value of x.

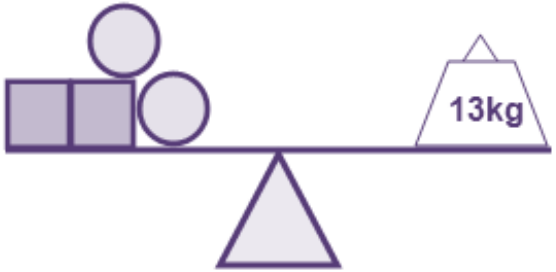
Substitute

Substitute your value of x to find the largest angle.

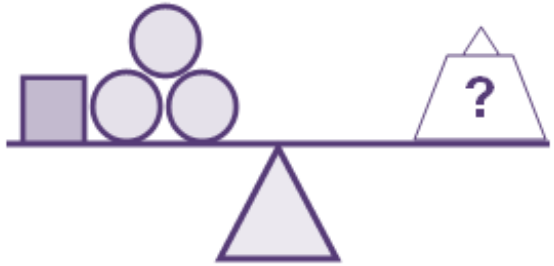
The Problem

Each of the following sets of scales is balanced.





What size weight is needed to balance this set?



My Solution

Some Help

First Scales First

From the first set of scales can you work out the weight of one of the squares?

Now the Second Scales

Now you know the weight of a square.
Use this to work out the weight of the two circles and then one of the circles.

Put them together

Now find the weight needed to balance the last set of scales. Use the information worked out earlier.

The Problem

You are given that

$$(a \diamond b) = 2a - b$$

e.g. $(5 \diamond 3) = 2 \times 5 - 3$

$= 7$

Use this information to solve

$$(2x \diamond x) = 21$$

Some Help

Understand what the symbol means?

Make sure you understand what the symbol means? Try using it with some numbers. Now try it with some different letters

Form an Equation

Now you understand what it means form an equation involving x.

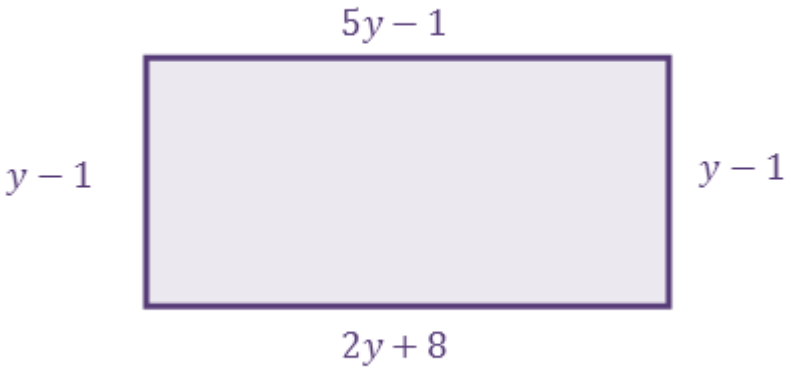
Solve

Solve your equation to find x.

My Solution

The Problem

A rectangle has the following lengths



Find the perimeter of the rectangle.

My Solution

Some Help

What do you know?

Look at the top and bottom sides of the rectangle. What do you know about these two sides?

Form an Equation and Solve

Use this fact to form an equation and solve your equation for y.

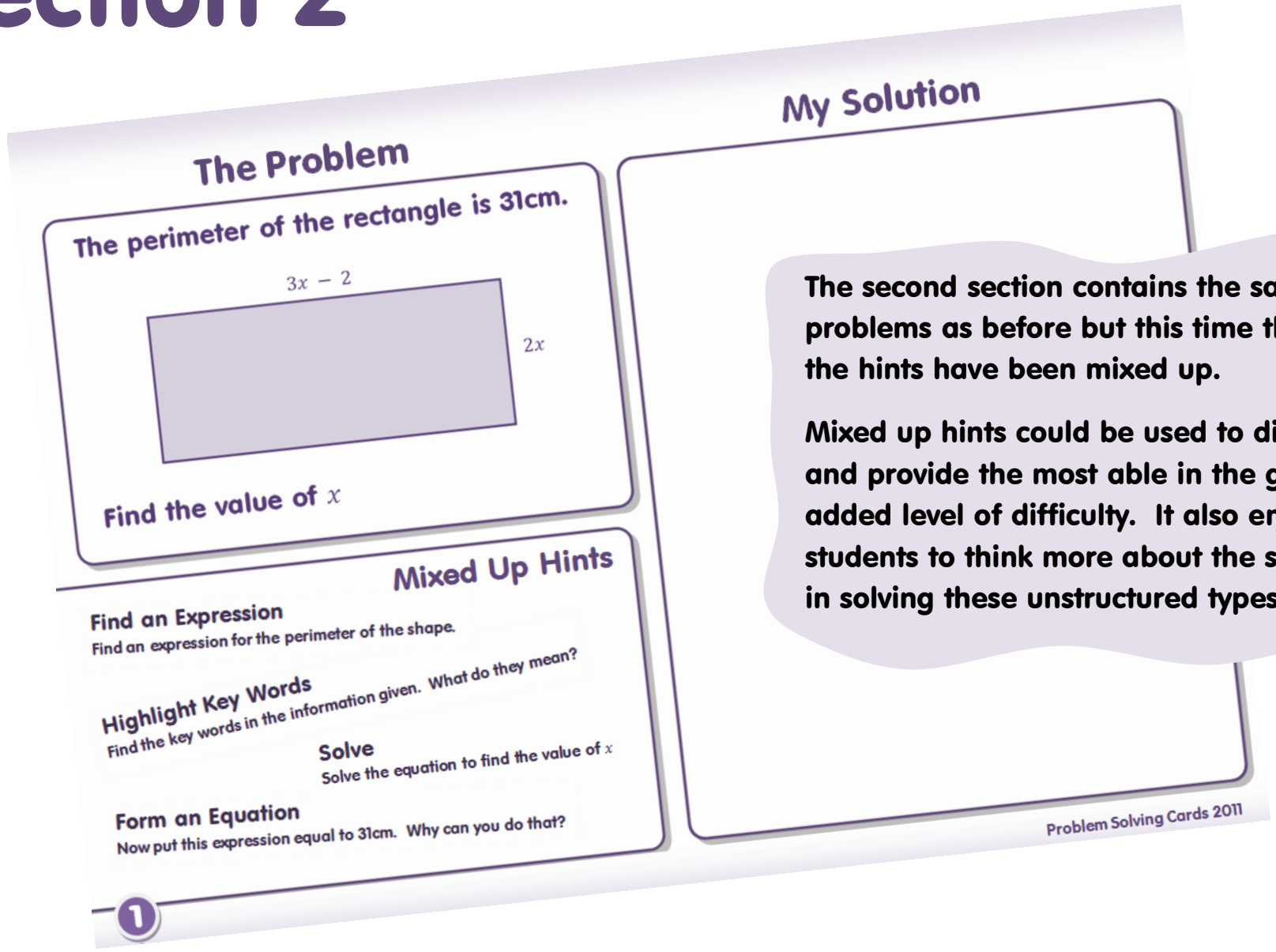
Form an Expression

Find an expression for the perimeter of the rectangle

Substitute

Substitute your value of y into the expression for the perimeter.

Section 2



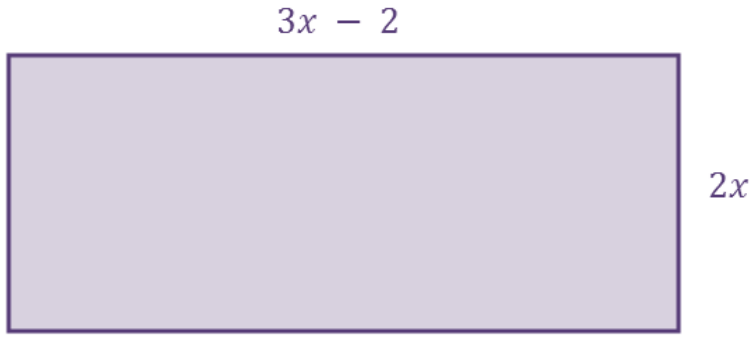
The second section contains the same multistep problems as before but this time the order of the hints have been mixed up.

Mixed up hints could be used to differentiate and provide the most able in the group with an added level of difficulty. It also encourages students to think more about the steps involved in solving these unstructured types of problems.

Problem & Mixed Up Hints

The Problem

The perimeter of the rectangle is 31cm.



Find the value of x

Mixed Up Hints

Find an Expression
Find an expression for the perimeter of the shape.

Highlight Key Words
Find the key words in the information given. What do they mean?

Solve
Solve the equation to find the value of x

Form an Equation
Now put this expression equal to 31cm. Why can you do that?

My Solution

The Problem

Every **row** of the following square adds up to 18

a	a	a
2b	a	a
a	b	2c

Work out the value of a + b + c

My Solution

Mixed Up Hints

Substitute

Substitute your values of a, b and c into the expression given

Now work out b and c

Use your value of a and what you know the 2nd row adds up to to work out the value of b? Use the 3rd row to work out c.

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What is the key piece of information you are told?

Work out a using the Top Row

What do you know the top row adds up to? What do you notice about the expressions in each of the boxes on this row? Now work out the value of a from this

The Problem

Here are two cards with expressions on them.

ab	$2a + b - c$

$a = 5$ and $b = 3$

Find the value of c so that the two cards are equal.

My Solution

Mixed Up Hints

Substitute Again

Substitute the values of a and b into the second expression. Can you work out the value of c now to make the cards equal?

Highlight the Key Information

What are the key pieces of information that you are given?

Substitute

Substitute the values of a and b into the expression on the first card. What does this tell you that the second card must equal?

The Problem

Adam, Barry and Charlie are brothers.

Here is some information about their ages

- Adam is twice as old as Barry
- Charlie is three years younger than Barry
- The sum of all their ages is 53

How old is Barry?

My Solution

Mixed Up Hints

Form an Equation and Solve
Put this expression equal to 53. Why can you do this? Now solve your equation.

Find an Expression
Find an expression for the sum of the brothers' ages.

Introduce some Algebra
Assume that Barry is b years old. Can you find expressions for the age of his two brothers Adam and Charlie in terms of b?

Highlight the Key Information
What are the key pieces of information that you are given?

The Problem

The first three diagrams in a sequence are shown.

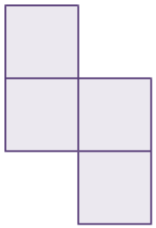

Diagram 1

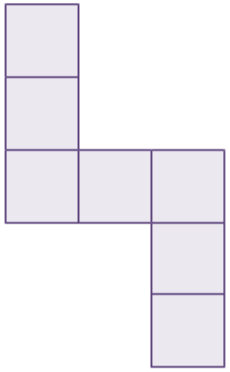

Diagram 2

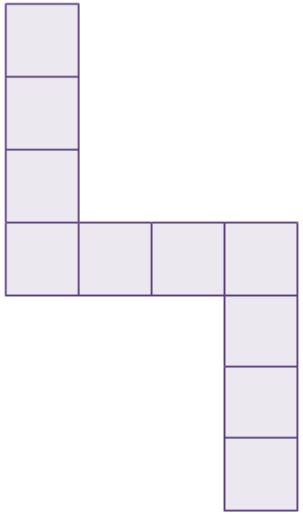

Diagram 3

How many squares will there be in Diagram 120?

My Solution

Mixed Up Hints

Substitute

Use your expression for the n^{th} term to find out how many squares will be needed for diagram 120. Think what you need to do with the number 120.

Put your Results into a Table

Make a table for your results. For each diagram write down how many squares make up the shape.

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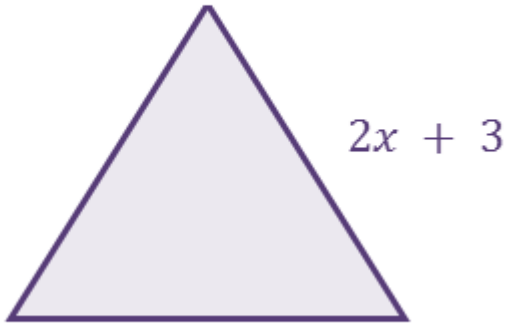


Find the value of x?

Mixed Up Hints

Highlight Key Words

Find the key words in the information given. What do they mean?

Solve

Solve the equation to find the value of x

Find an Expression

Find an expression for the perimeter of the shape.

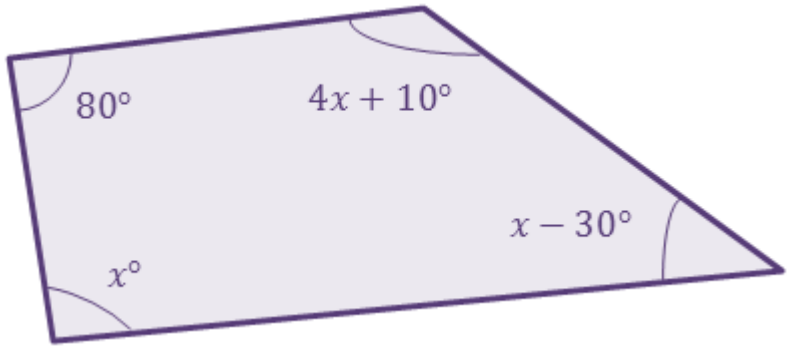
Form an Equation

Now put this expression equal to 42cm. Why can you do that?

My Solution

The Problem

The angles in a quadrilateral are shown.



Find the size of the largest angle.

My Solution

Mixed Up Hints

Substitute

Substitute your value of x to find the largest angle.

Solve your Equation

Solve the equation to find the value of x.

Find an Expression

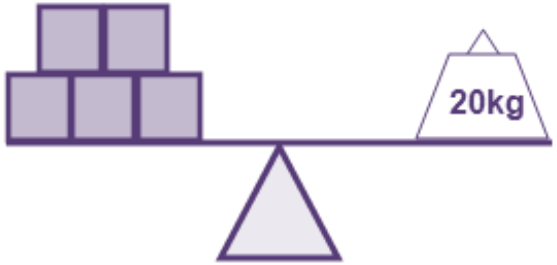
Find an expression for the sum of all the angles.

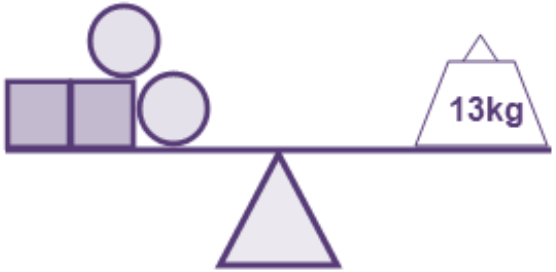
Form an Equation

What can you put this expression equal to? Think about what the angles in a quadrilateral add up to.

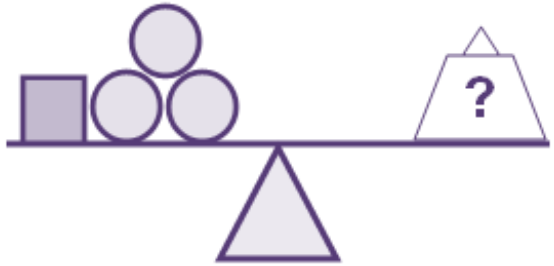
The Problem

Each of the following sets of scales is balanced.





What size weight is needed to balance this set?



My Solution

Mixed Up Hints

Put them together

Now find the weight needed to balance the last set of scales. Use the information worked out earlier.

Now the Second Scales

Now you know the weight of a square.
Use this to work out the weight of the two circles and then one of the circles.

First Scales First

From the first set of scales can you work out the weight of one of the squares?

The Problem

You are given that

$$(a \diamond b) = 2a - b$$

e.g. $(5 \diamond 3) = 2 \times 5 - 3$

$= 7$

Use this information to solve

$$(2x \diamond x) = 21$$

Mixed Up Hints

Form an Equation

Now you understand what it means form an equation involving x.

Understand what the symbol means?

Make sure you understand what the symbol means? Try using it with some numbers. Now try it with some different letters

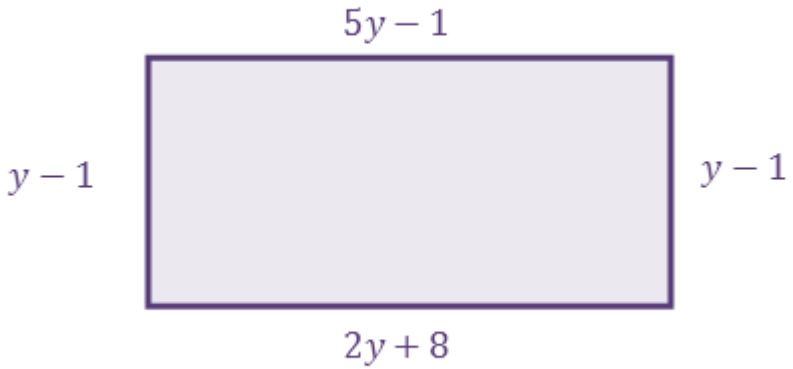
Solve

Solve your equation to find x.

My Solution

The Problem

A rectangle has the following lengths



Find the perimeter of the rectangle.

Mixed Up Hints

Form an Expression

Find an expression for the perimeter of the rectangle

What do you know?

Look at the top and bottom sides of the rectangle. What do you know about these two sides?

Form an Equation and Solve

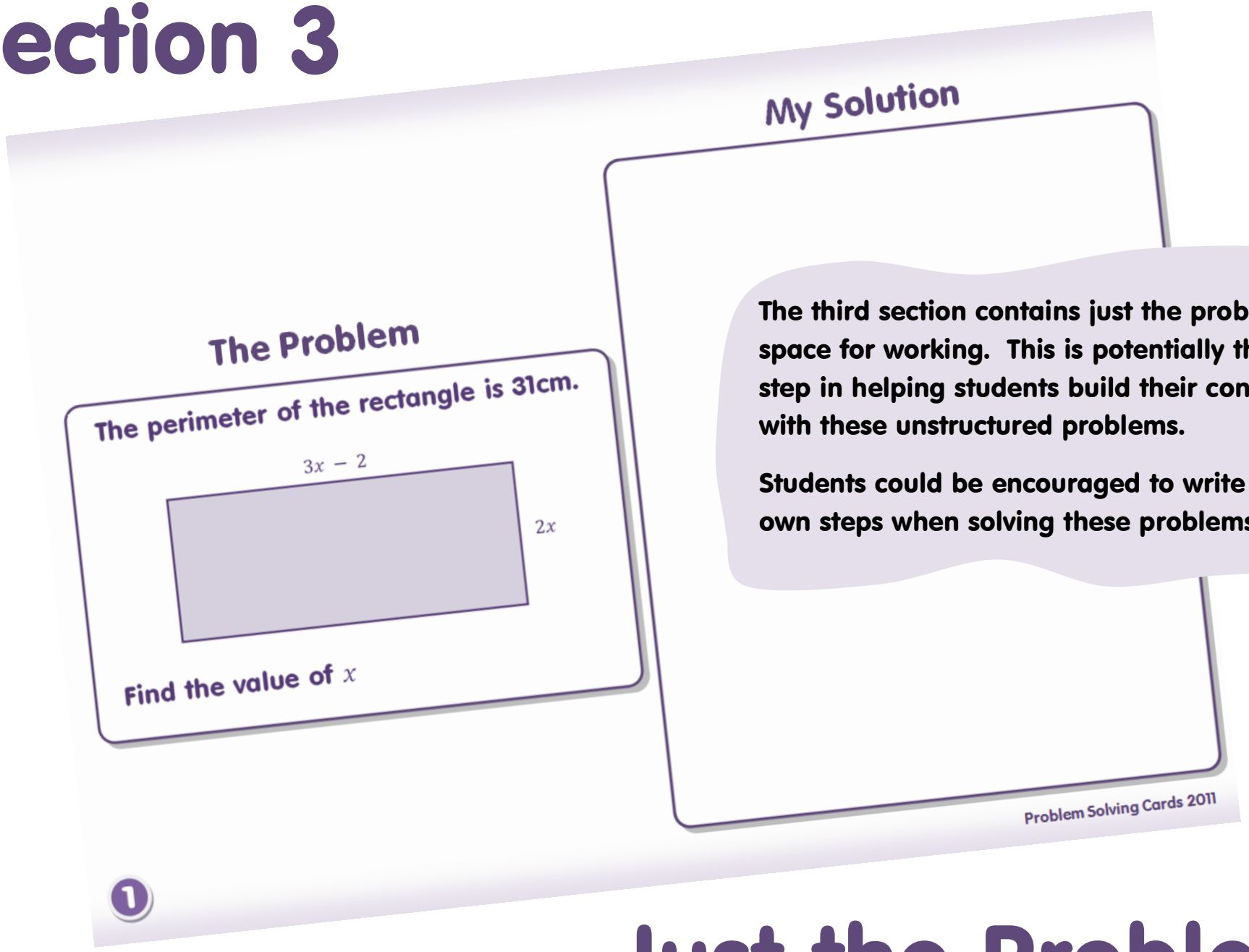
Use this fact to form an equation and solve your equation for y.

Substitute

Substitute your value of y into the expression for the perimeter.

My Solution

Section 3



The third section contains just the problems and space for working. This is potentially the next step in helping students build their confidence with these unstructured problems.

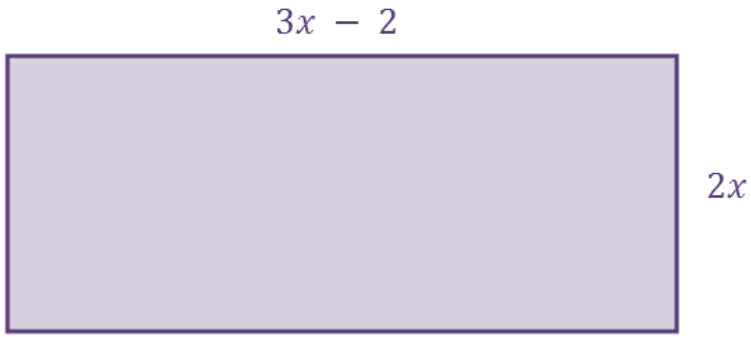
Students could be encouraged to write out their own steps when solving these problems.

Just the Problems

My Solution

The Problem

The perimeter of the rectangle is 31cm.



Find the value of x

1

The Problem

Every **row** of the following square adds up to 18

a	a	a
2b	a	a
a	b	2c

Work out the value of a + b + c

My Solution

The Problem

Here are two cards with expressions on them.

ab	$2a + b - c$

$a = 5$ and $b = 3$

Find the value of c so that the two cards are equal.

My Solution

My Solution

The Problem

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How old is Barry?

The Problem

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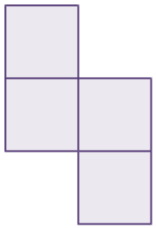

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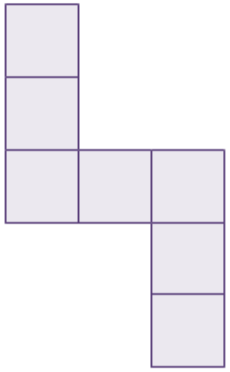
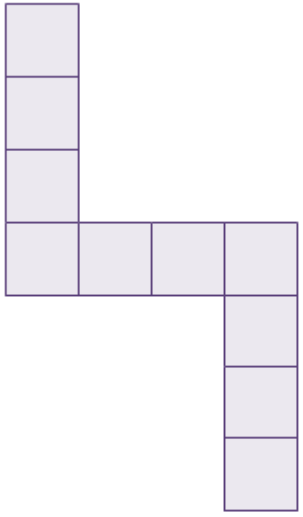


Diagram 2



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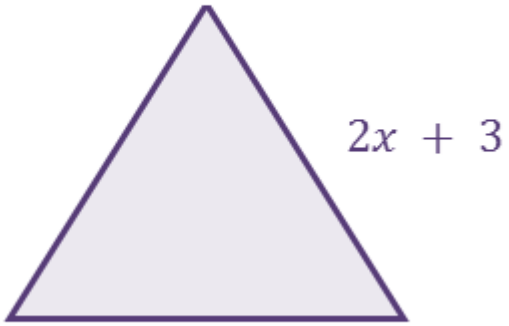

Find the value of x?

My Solution

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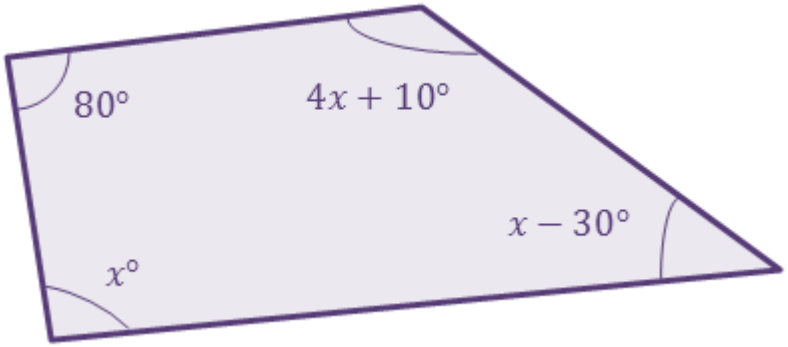

Find the size of the largest angle.

My Solution

The Problem

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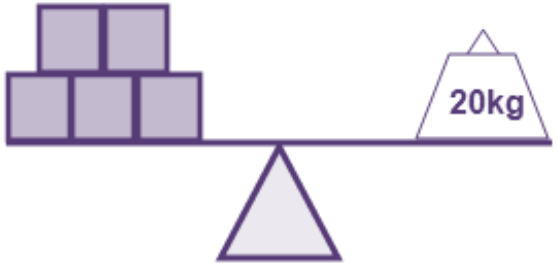

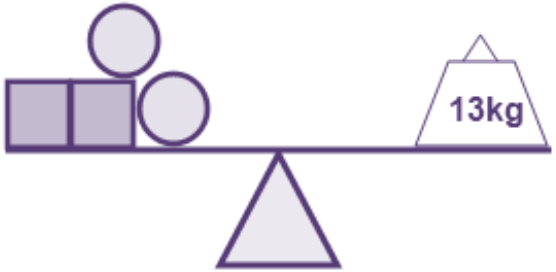

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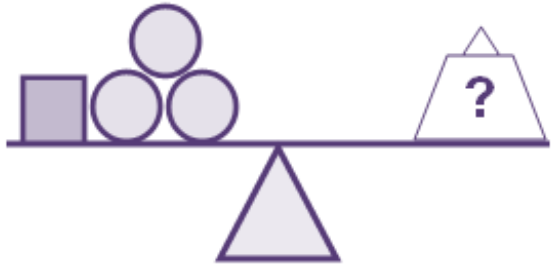

My Solution

My Solution

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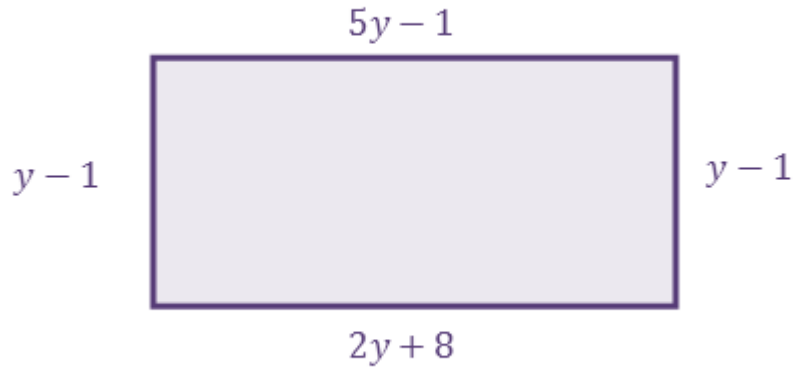
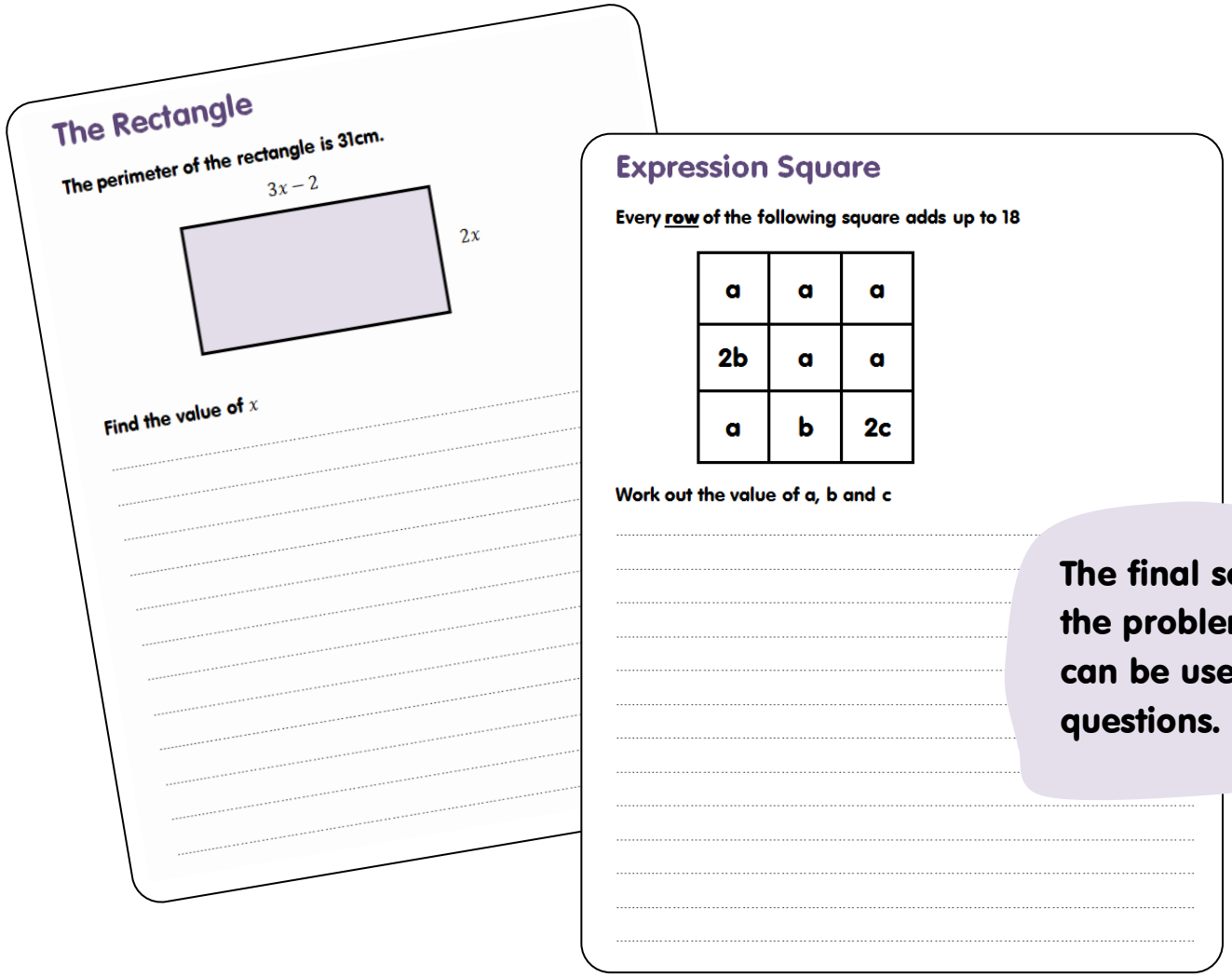


Find the perimeter of the rectangle.

My Solution

Section 4



The Rectangle

The perimeter of the rectangle is 31cm.

$3x - 2$

$2x$

Find the value of x

Expression Square

Every **row** of the following square adds up to 18

a	a	a
2b	a	a
a	b	2c

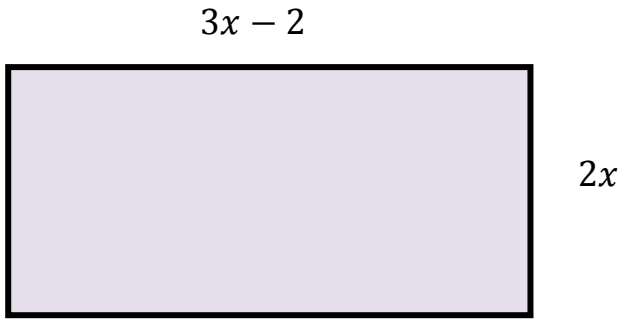
Work out the value of a, b and c

The final section contains some variations on the problems presented in this booklet. These can be used as extension or additional questions.

Variations

The Rectangle

The perimeter of the rectangle is 31cm.



Find the value of x

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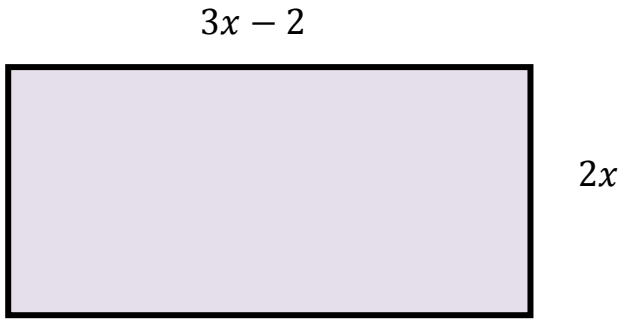
The Rectangle

The perimeter of the rectangle is 31cm.



Find the area of the rectangle

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Expression Square

Every <u>row</u> of the following square adds up to 18

a	a	a
2b	a	a
a	b	2c

Work out the value of a, b and c

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Expression Square

Every row of the following square adds up to 18

a	a	a
2b	a	a
a	b	2c

Work out the value of a + b + c

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Expression Cards

Here are two cards with expressions on them.

ab	$2a + b + c$

$a = 5$ and $b = 3$

Find the value of c so that the two cards are equal.

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Expression Cards

Here are two cards with expressions on them.

ab	**2a + b - c**

a = 5 and b = 3

Find the value of c so that the two cards are equal.

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Three Brothers

Adam, Barry and Charlie are brothers.

Here is some information about their ages

- **Adam is twice as old as Barry**
- **Charlie is three years younger than Barry**
- **The sum of all their ages is 53**

How old is Barry?

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Three Brothers

Adam, Barry and Charlie are brothers.

Here is some information about their ages

- **Adam is twice as old as Barry**
- **Charlie is three years younger than Barry**
- **The sum of all their ages is 53**

How many years older is Adam than Charlie?

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Sequence Problem

The first three diagrams in a sequence are shown.

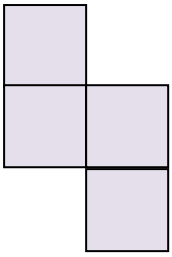

Diagram 1

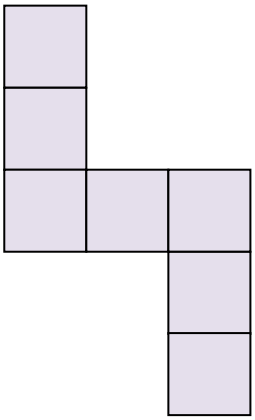

Diagram 2

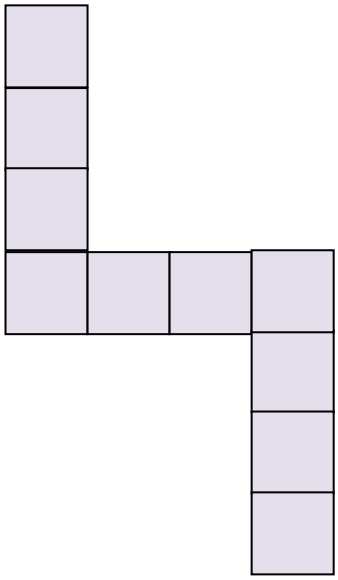

Diagram 3

How many squares will there be in Diagram 120?

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Sequence Problem

The first three diagrams in a sequence are shown.

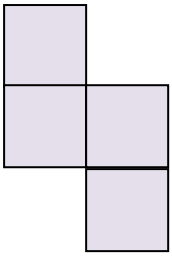

Diagram 1

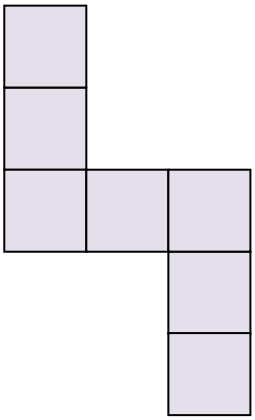

Diagram 2

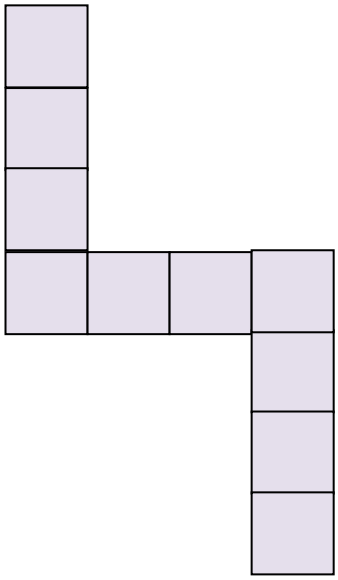

Diagram 3

Which diagram will contain 1057 squares?

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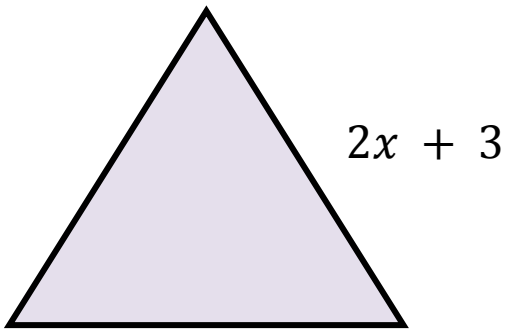
Equilateral Triangle

The diagram shows an equilateral triangle.

The perimeter of the triangle is 42cm.



Find the value of x?

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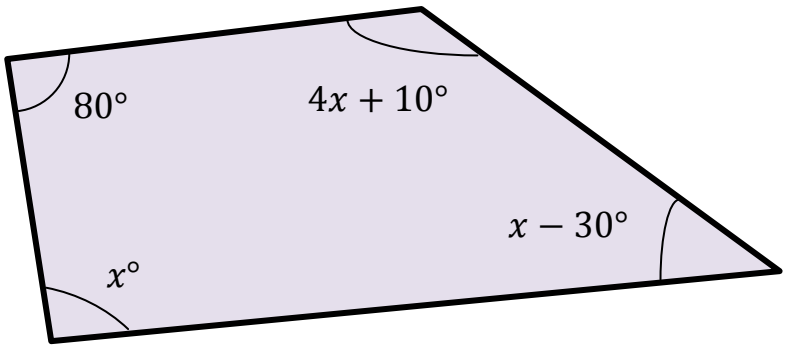
Quadrilateral

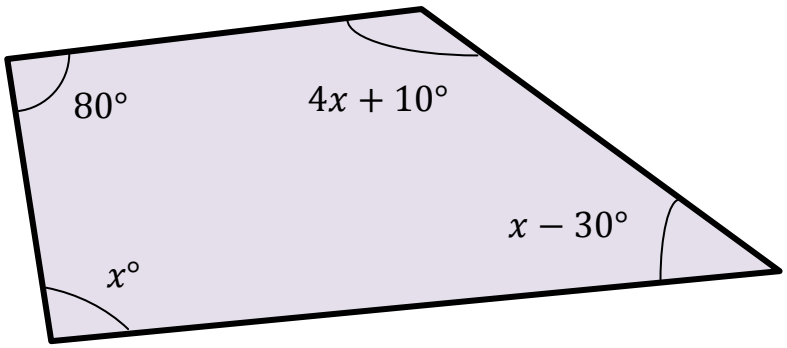
The angles in a quadrilateral are shown.



Find the value of x.

Quadrilateral

The angles in a quadrilateral are shown.



Find the size of the largest angle.

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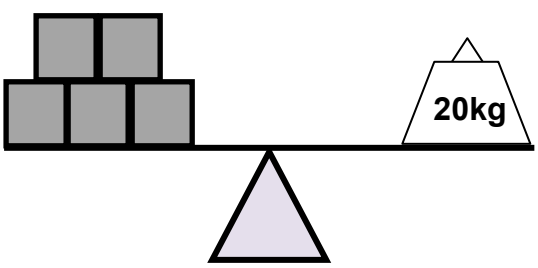
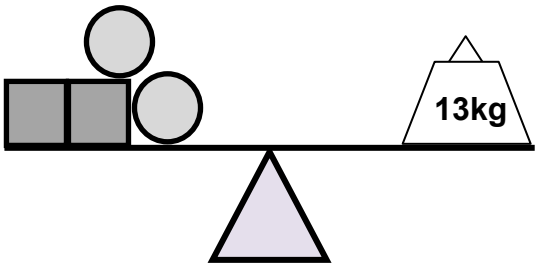
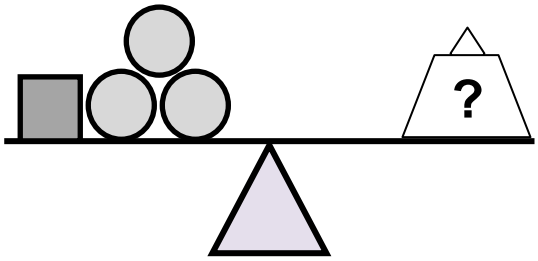
Scales

Each of the following sets of scales is balanced.





What size weight is needed to balance this set?



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The Symbol

You are given that

$$(a \diamondsuit b) = 2a - b$$

e.g. $(5 \diamondsuit 3) = 2 \times 5 - 3$

$= 7$

Use this information to solve

$$(5x \diamondsuit x) = 27$$

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The Symbol

You are given that

$$(a \diamond b) = 2a - b$$

e.g. $(5 \diamond 3) = 2 \times 5 - 3$

$= 7$

Use this information to solve

$$(4x \diamond 2x - 3) = 27$$

The Symbol

You are given that

$$(a \diamond b) = 2a - b$$

e.g. $(5 \diamond 3) = 2 \times 5 - 3$

$= 7$

Use this information to solve

$$(3x \diamond 2x) = (x \diamond 4)$$

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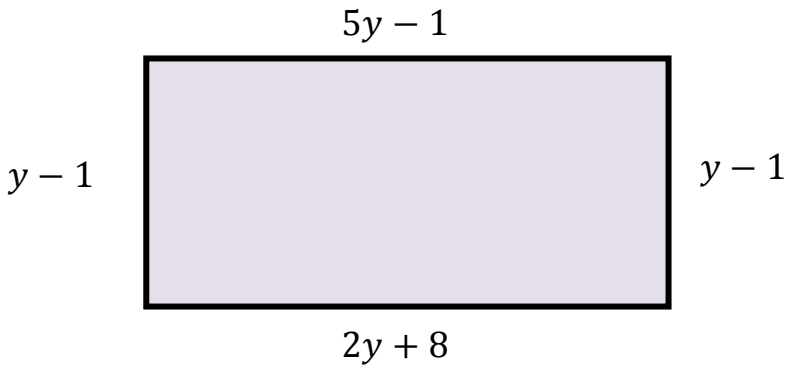
Rectangle 2

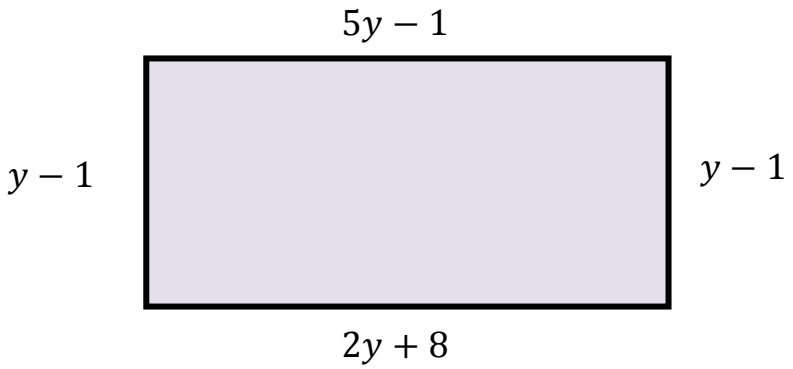
A rectangle has the following lengths



Find the value of y

Rectangle 2

A rectangle has the following lengths



Find the perimeter of the rectangle.

Answers

The following are the answers to the 10 problems included

1. $x = 3.5$

2. $a = 6, b = 3, c = 4.5 \; therefore \; a + b + c = 13.5$

3. $c = -2$

4. $Barry \; is \; 14 \; years \; old$

5. $361 \; squares$

6. $x = 5.5$

7. $193°$

8. $11.5kg$

9. $x = 7$

10. $y = 3 \; therefore \; perimeter \; is \; 16 \; units$

Please pass on any comments and suggestions. Feel free to use as you see fit with your classes. More to follow. Hope you enjoy using the problems. Pencho @ TES